CHAPTER 10: Introduction to Mathematical Finance



Objectives:

- a. Understand various entries in a credit card statement
- **b.** Differentiate stocks and basic of bonds
- c. Compute finance charge and new balance in a credit card loan

Lesson 1: Consumer Loans

Installment Loans

- is a type of agreement or contract involving a loan that is repaid over time with a set number of scheduled payments.
 - **Cash price** the price of the merchandise that a person is buying.
 - **Down payment** if the person pays part of the cash price at the time of purchase.
 - **Amount Financed** the balance is usually paid off in equal monthly installments.

Example:

Mary purchased a refrigerator that costs Php 6,750. She made a 20% down payment and paid Php 390 a month payable in 10 months. Find the total amount that she paid for the refrigerator.

Solution:

From the information, we know the down payment is

Down payment = 0.20 × 6,750 = Php 1,350

Next, the total amount of the monthly payment can be computed as

Total of monthly payments = 10×390 = Php 3,900

Hence, the total amount that Mary must pay for the refrigerator is

Php 1,350 + Php 3,900 = Php 5,250

Thus, Mary paid Php 5,250 for a refrigerator that cost Php 6,750, so the extra charge or finance charge was

The amount of the monthly payment can also be computed. The procedure is shown in the next example.

Example:

Felipe purchased an LCD television set for Php 15,600. He made a 15% down payment and paid the balance over 18 months. The store charged 8% interest on the amount financed. Find the monthly payment.

Solution:

Let us find the amount financed.

Down payment = 0.15 × Php 15,600 = Php 2,340

Amount financed = Php 15,600 – Php 2,340 = Php 13,260

Now, let us find the interest for the remaining amount to be paid in 18 months. In this case, the interest rate is 8% and 18 months is 1.5 years.

 $I = Prt = Php 13,260 \times 0.08 \times 1.5 = Php 1,591.20$

Now, let us add the payment amount financed and the interest, then divide the sum by 18 months.

Monthly payments = (13,260 + 1,591.20) ÷ 18

≈ Php 825.10

Hence, the monthly payment is Php 825.10.

Annual Percentage Rate

The most common, if not universal way to express the amount of interest to be paid on a loan is the *annual percentage rate* (*APR*). The interest is expressed as a percentage or a fraction of the amount of money loaned if the money were to be loaned, with no intermediate payments or corrections, for a year. The Truth in Lending Act requires all lending institutions to tell the borrower the true percent of interest. The true percent is called the *APR*.

In order to find the *APR*, a table is issued by the Monetary Board and can be used by lending institutions. The table is quite lengthy and so a mathematical formula can be used to find the *approximate APR*. If payments are made monthly, the formula is

$$APR = \frac{24 \times \text{total interest}}{\text{Principal} \times (1 + \text{total number of payments})} = \frac{24I}{P(1+T)}$$

Where

I = Total interest P = principal T = total number of payments

Example:

If a person borrowed Php 8,000 payable in 2 years and paid 8% per year, find the *APR* for the loan if monthly payments were made.

Solution:

First, find the total amount of interest.

 $I = Prt = 800 \times 0.08 \times 2 = Php 1,280$ In this case, we have I = Php 1,280 P = Php 8,000, and T = 12 monthly payments $\times 2$ years = 24 $APR = \frac{24 \times 1,280}{8,000 \times (1+24)}$

= 0.1536 or 15.36%

Demand Loans

- Is a loan for which repayment, in full or in part, may be required at any time, or made at any time. **Demand Note** is the financial instrument representing a demand loan.

Example:

Mr. Ortiz borrowed Php 120,000 from the Royal Bank on a demand note. He agreed to repay the loan in the six equal monthly installments (each payment is made at the end of the month), and also authorized the bank to collect interest monthly from its bank account at an APR of 6% calculated on the unpaid balance. What will the loan coast?

Solution:

From the given information, we have

Monthly payment of principal = $120,000 \div 6 = Php 20,000$

Monthly rate of interest $=\frac{6\%}{12}=0.5\%=0.005$

| Month | Loan Amount Owed During Month | Interest Collected for Month | |
|-----------|----------------------------------|------------------------------------|----------------------|
| 1 | Php 120,000 Original | Php 600 | (120,000)(0.005) |
| 2 | Php 100,000120,000 - 20,000 | Php 500 | (100,000)(0.005) |
| 3 | Php 80,000 100,000 - 20,000 | Php 400 | (80,000)(0.005) |
| 4 | Php 60,000 80,000 - 20,000 | Php 300 | (60,000)(0.005) |
| 5 | Php 0,000 60,000 - 20,000 | Php 200 | (40,000)(0.005) |
| 6 | Php 0,000 40,000 - 20,000 | Php 100 | (20,000)(0.005) |
| Total int | terest cost | → Php 2,100 | |

Repayment of Loan

An arrangement happens when a lender gives money or property to a borrower, and the borrower agrees to return the property or repay the money, usually along with interest, at some future point(s) in time. Most people are familiar with the repayment of loans. The repayment of loans is best illustrated by the example that follows:

Example:

Roberto borrows Php 100,000 now at an interest rate of 8 percent per annum. The loan has to be repaid through five equal installments after each of the next five years. What is the annual repayment?

Solution:

Let us start by calculating the final value of the loan (at the end of 5th year) by using the formula

$$A = P(1+i)^n$$

Where P = 100,000 i = 8% = 0.08 n = 5 and S = the sum invested after 5 years

Hence,

 $S = 100,000(1 + 0.08)^5 = 146,932.81$

The value of the initial loan after 5 years (Php 146,932.81) must equal the sum of the repayments. Since the sum of repayments must equal the final value of the loan (that is Php 146,932.81). Let us apply the formula:

$$S = R \cdot s_{n i i}$$

$$S = R \cdot \left[\frac{(1+i)^n - 1}{i}\right]$$

$$146,932.81 = R \cdot \left[\frac{1.08^5 - 1}{1.08 - 1}\right]$$

It follows that R = 25,045.65

Thus, the annual repayment is Php 25,045.65.

Rule of 78s

Some installment loans can be paid off early. When this occurs, the borrower can save money on the interest. For example, if a 1-year installment loan is paid off in 6 months, you may think that you should pay one-half of the interest since you are using the money for half of the term. However, in many institutions, this is not the case.

Lending institutions use what is called the *rule of 78s* to compute the interest owed when a loan is paid off early.

The rule of 78s requires that the larger amounts of the interest be paid with the earlier payments. Remember though that payments are the same amount each month. The rule of 78s for a 12-month loan requires that

¹²/₇₈ of the interest be paid on the first month.
¹¹/₇₈ of the interest be paid on the second month.
¹⁰/₇₈ of the interest be paid on the third month.
⁹/₇₈ of the interest be paid on the fourth month.
¹/₇₈ of the interest be paid on the last (twelfth) month.

(When we add $\frac{12}{78} + \frac{11}{78} + \frac{10}{78} + \dots + \frac{2}{78} + \frac{1}{78}$, the sum is 1.)

When loans are secured for longer or shorter periods of time, the rule of 8s is adjusted accordingly as shown in the examples.

The formula that is used to find the refund when a loan is paid off early is

Refund = total interest \times refund factor. The refund factor is found by this solution below:

Refund factor = $\frac{\text{sum of number of months remaining}}{\text{sum of number of months of loan}}$

Example:

An installment loan for 12 months was paid off at the end of the seventh month. Find the refund factor.

Solution:

Since the loan is paid off at the end of the seventh month, there are five months remaining (i.e, 12-7=5), so the numerator of the refund factor is 1 + 2 + 3 + 4 + 5 = 15. The denominator is the sum of the numbers for the entire loan, i.e., 12-month. Hence it is 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 = 78. The refund factor is $\frac{15}{78}$.

Example:

An installment loan for 10 months is paid off at the end of the third month. Find the refund factor.

Solution:

The duration of the loan is 10 months, so the sum of the digits for the denominator is

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$$

Since the loan was paid off at the end of the third month, there are 10-3=7 months remaining. Hence, the sum of the numbers of the numerator is

$$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$$

The refund factor is $\frac{22}{55}$.

Remark: rather than adding a sequence of consecutive numbers starting from 1 to *n*, a mathematical shortcut can be used. It is $Sum = \frac{n(n+1)}{2}$.

For example, if you want to add the consecutive numbers from 1 to 30, n=30 and so the sum is

$$\frac{30(30+1)}{2} = \frac{30(31)}{2} = \frac{930}{2} = 465$$



For more knowledge about Consumer Loan, please check the link provided; <u>https://investinganswers.com/dictionary/d/demand-loan</u> <u>https://www.investopedia.com/terms/r/ruleof78.asp</u>

REMEMBER





ACTIVITY:

Read the question and solve the problem below. Write

your complete solution.

Mike Marquez purchased a lawn tractor for Php 9,000. He made a down payment of Php 1,000 and financed the rest at 7% payable in 24 months. He paid off the loan at the end of the fifteenth months. Find the amount of his refund using the rule of 78s.

Lesson 2: Credit Cards

"Buy now, pay later" is a concept that has become a common way by which individuals and businesses purchase goods and services. Merchants in all categories and lending institutions alike, encourage us to just say "charge it!".

Line of Credit

Is a pre-approved loan agreement between a financial institution and a borrower. The borrower may withdraw money up to an agreed maximum, at any time. Interest is charged only on the amount withdrawn from the line of credit. A minimum repayment may be required each month. The borrower may repay any additional amount at any time without further penalty. The rate of interest charge for money borrowed on a line of credit is often lower than the rate of interest charged on most credit cards. The interest rate may change over time.

Credit Card

Is a card entitling the bearer to a revolving line of credit with a pre-established credit limit. Interest rates which are set by the credit card issuers, very considerably, and can be charged at any time. Generally, interest rates charged on credit cards are higher than rates charged on loans made by financial institutions.





Finance charge

- Credit card interest.

Credit Card Statement

The credit card statement given monthly to the cardholder indicates the previous balance, finance charge, total purchases, any payments and credits, and new balance. If the cardholder did not pay his/her previous bill in full, the credit card or charge account company adds a *finance charge*. This is interest that the company charges to a credit account for not paying the total amount owed by the due date. The summary line on the credit card statement uses this formula to determine your new balance.

 $\frac{\text{New}}{\text{Balance}} = \binom{\text{Previous}}{\text{Balance}} + \binom{\text{Finance}}{\text{Charge}} + \binom{\text{New}}{\text{Purchases}} - (\text{Payments} + \text{Credits})$

| SAMPLE CREDIT CARD STATEMENT Account Number | | | | | | umber |
|---|----------|---|---|--|-----------------------------|--------------------|
| Make check pay | able to | o : | | Due Date 7/04/02 | Minimum Due \$31.00 | |
| My Credit Card P. O. Box 4567 Anywhere, CA 91111 | | | | | New Balance | Amount Enclosed |
| Return top portio | n with | payment | | \$1379.90 | | |
| Reference Posting Des Number Date Tra | | Description Transact | on of ion | Debits | Debits Credits a Payment | |
| 0077623 0608 ABC Stores 0317886 0608 Autopay Cat 7075061 0610 Let's Go Cat 4871311 0610 Sleep Nice 1 5887041 0622 AAA Airlines 9283742 0630 payment - th | | Unlimited ble Network r Rental Motel s thank you | 108.20 58.00 159.72 128.00 109.32 | | 40.00 | |
| Previous Balance - Payments 839.62 40.00 | | - Payments 40.00 | + New Charges 563.24 | Finance Charge 17.04 | = New 8 1379. | Balance 90 |
| Average Daily Balance | Mo Ra | onthly Periodic ate | Annual Percentage Rate | Finance Charge | Credit L \$3,000. | imit 00 |
| 1011.00 | 1.0 | 042% | 12.50 | 17.04 | Credit A \$1620.1 | vailable 0 |

Example:

Review the sample credit card statement above. What is his new balance?

Solution:

To find the new balance, use the formula:

 $\frac{\text{New}}{\text{Balance}} = \binom{\text{Previous}}{\text{Balance}} + \binom{\text{Finance}}{\text{Charge}} + \binom{\text{New}}{\text{Purchases}} - (\text{Payments} + \text{Credits})$ $\frac{\text{New}}{\text{Balance}} = (839.62) + (17.04) + (563.24) - (40)$ $\frac{\text{New}}{\text{Balance}} = 1379.90$

Calculation of Finance Charge and New Balance in the Credit Card

Example:

Bernard's Chocolate Shop has a Php 200,000 line of credit with the Western Bank. The *APR* charged on the account is the current prime rate plus 4%. There is a minimum APR on the account of 10%. The starting balance on April 1 was Php 23,500. On April 9, Bernard borrowed Php 15,000 to pay for a shipment of assorted gift items. On April 20, he made a Php 30,000 payment on the account. On April 26, another Php 25,000 was borrowed to pay for air conditioning repairs. The billing cycle for April has 30 days. If the current rate is 8%, what is the finance charge on the account and what is Bernard's new balance?

Solution:

To solve this problem, we must find the *APR*, the periodic rate, the average daily balance, the finance charge and finally the new balance.

| Dates | Number of Days | Activity/ Amount | Unpaid Balance | Daily Balance (Unpaid balance × days) |
|-------------|-------------------|---|---------------------|--|
| April 1-8 | 8 | Previous balance | Php 23,500 | Php 188,000 |
| April 9-19 | 11 | Borrowed Php 15,000 | Php 38,500 | Php 423,500 |
| April 20-25 | 6 | Payment Php 30,000 | Php 8,500 | Php 51,000 |
| April 26-30 | 5 | Borrowed Php 25,000 30 days in cycle | Php 33,500 Total | Php 167,500 Php 830,000 |

| Average daily balance = | Sum of daily balances | _ | Php 830,000 | \sim Dhp 2766667 |
|-------------------------|-----------------------|---|-------------|--------------------|
| | Days in billing cycle | _ | 30 | ~ Plip 27,000.07 |

Finance charge = Average daily balance × Periodic rate

Finance charge = Php $27,666.67 \times 0.08$ = Php 2,213.33

 $\frac{\text{New}}{\text{Balance}} = \binom{\text{Previous}}{\text{Balance}} + \binom{\text{Finance}}{\text{Charge}} + \binom{\text{Loan}}{\text{Amount}} - (\text{Payments})$ $\frac{\text{New}}{\text{Balance}} = \text{Php } 23,500 + \text{Php } 2,213.33 + \text{Php } 40,000 - \text{Php } 30,000$ $\frac{\text{New}}{\text{Balance}} = \text{Php } 35,713.33$

Finance Charge Using the Unpaid Balance Method

Example:

Refer to the table below for the summary portion of Armando Reyes charge account statement. He had a previous balance of Php 13,240, made a Php 400 payment, and purchased an item for Php 795.50. the monthly finance charge is computed at 1.5% of the unpaid balance. Determine the (a) unpaid balance, (b) finance charge and (c) new balance.

| Charge Account Statement | | | | | | | |
|--------------------------|-------------------|---------|---------|------------|---------|--|--|
| 88 | Payment/Thank You | | 400 | | | | |
| Billing Date: | | | | | | | |
| 2/16 | | | | | | | |
| Previous | Payments & | Unpaid | Finance | New | New | | |
| Balance | Credits | Balance | Charge | Purchases | Balance | | |
| Php 13,240.00 | Php 400.00 | a. | b. | Php 795.50 | С. | | |

Solution:

a. Unpaid Balance = Previous Balance – (Payments + Credits)

Unpaid Balance = Php 13,240 – Php 400 = Php 12,840

b. Finance Charge = Unpaid Balance × Periodic Rate

Finance Charge = Php $12,840 \times 1.5\%$ = Php 192.60

c. New Balance = Unpaid Balance + Finance Charge + New Purchases

New Balance = Php 12,840 + Php 192.60 + Php 95.50 = Php 13,828.10



For more knowledge about Finance Charge and New Balance in Credit Card, please check the link provided; <u>https://www.chegg.com/homework-help/find-finance-charge-new-balance-credit-card-account-previous-chapter-13.2-problem-4p-solution-9780495388838-exc</u>

REMEMBER



ACTIVITY:

Read the question and solve the problem below. Write

your complete solution.

Refer to the table below for the summary portion of Joe Saavedra charge account statement. He had a previous balance of Php 112,00 made a Php 3,000 payment, and purchased an item for Php 1,500. The monthly finance charge is computed at 2.5% of the unpaid balance. Determine the (a) unpaid balance, (b) finance charge and (c) new balance.

| Charge Account Statement | | | | | | | |
|--------------------------|-------------------|---------|---------|-----------|---------|--|--|
| 88 | Payment/Thank You | | 400 | | | | |
| Billing Date: | | | | | | | |
| 2/16 | | | | | | | |
| Previous | Payments & | Unpaid | Finance | New | New | | |
| Balance | Credits | Balance | Charge | Purchases | Balance | | |
| Php 112,000 | Php 3,000.00 | a. | b. | Php 1,500 | C. | | |

Lesson 3: Stocks and Bond Basics

Stocks

- Otherwise known as equity securities, are the most popular and known to be most exciting in the field of finance and investment.
- The term stock is generally defined as representing a share or portion of a company. When an investor purchases shares or stock, he or she becomes a part of the company.
- The investor is called the *shareholder*.

Example:

If a company issues 10,000 shares of stock and an investor purchases 2,000 shares, then the investor owns $\frac{1}{5}$ of the company.

Stocks bonds

How does a Stock differ from Bond?

A major difference between stocks and bonds is the type of investment each is classified.

A **stock** is an equity while **bonds** are debt. In other words, a shareholder is an owner or investor in a firm, while a bondholder is a creditor to the firm.

Each of these two forms of investment maintains different rights, risks and return dynamics.

Bondholder's rights are confined by contract to certain obligations of the business in connection with their bond. Usually, it is the amount of interest, the duration of the payments, or any specific commitment a firm may make on that particular bond that bondholders are paid in. Bondholder's rights, in other words, are traditionally set out in contract and not through common law. As for shareholders of a company, their rights are limited to a voting power in the firm's decisions.

The voting power can be used in number of scenarios. The most common situation is that during a shareholders meeting, the investors with voting rights elect the board of directors, and usually vote on a variety of other issues. These other issues may include mergers and acquisitions, major decisions where directors may have a conflict of interest, and making nonbinding recommendations about governance and management of the company to the board of directors.

Being affected by the business' financial health and ability to survive. The risk for a common shareholder in the case of bankruptcy is relatively high, as they are often the last in line during liquidation. However, debt holders, specifically secured debt holders, are usually the first asset holders in line (after lawyer and liquidation fees of course). They will be the first to receive new securities, cash, or a mixture of both, in the case of reorganization, for example. Simply put, creditors have less downside risk than equity holders, while equity holders have a higher potential to return.

More on Stocks and Dividends

• **Par Value –** the price of the share of a stock.

Par Value is set by the company when the stock is first sold to the public. When the stock is resold in the stock market, its price is determined by what the buyer is willing to buy, and what the seller is willing to accept. This value, called the *market value*, is published in the business or financial section of most major newspapers and online. In order to get information about certain stock, we can refer to the table below.



Example:

Refer to table above, use the information posted for the AU Bank to answer the following questions.

- a. What is the highest stock selling price in the past 52 weeks?
- b. What is the lowest stock selling price in the past 52 weeks?
- c. What is the amount of dividend per share that AU Bank paid to its shareholders last year?
- d. If a person owned 250 shares of stock, how much did the person make in dividends last year?
- e. How many shares were traded yesterday?
- f. What was the closing price per share yesterday?

Solution:

- a. Php 75.30
- b. Php 66.00

- c. Php 5.05
- d. $250 \times Php 5.05 = Php 1,262.50$
- e. $56,610 \times 100 = 5,661,000$ shares
- f. Php 70.80

Valuation of Stocks

Corporations are built and expanded with money known as capital.

Capital is raised by issuing and selling shares of stocks. Investors' ownership in a company is measured by the number of shares they own. Each ownership portion, or share, is represented by a *stock certificate*.



Two classes of stock to appeal to different types of investors.

- 1. Common stock
- 2. Preferred stock
- Common Stock
- An investor shares directly in the success or failure of the business. When the company does well, the dividends and price of the stock may rise, and the investors

make money. When the company does poorly, it does not pay dividends and the price of the stock may fall.

Preferred Stock

The dividends are fixed, regardless of how the company is doing. When the board of directors of a company declare a dividend, the preferred stockholders are paid before the common. If the company goes out of business, the preferred stockholders have priority over the common as far as possibly getting back some of their investment. Most preferred stock is like a perpetuity because it has no future maturity value. The value or price is like A_{∞} , the dividend is like the rent *R*, and the yields rate is the *i*. The equation for both preferred stock and common stock if they pay a fixed value (or current dividend per share, assuming no growth) is given by this:

$$V = \frac{D}{k}$$

Where D is the annual dividend per share, and k is the yield rate.

Therefore, k (or yield) = $\frac{D}{V} = \frac{Annual \, dividend \, per \, share}{Current \, price \, of \, one \, share}$

Compute this to the perpetuity formula we have introduced

$$A_{\infty} = \frac{R}{i}$$

Example:

Cristina owns common stock in ABC Corp. the annual dividend is Php 3.40. the current price is Php 67.40 per share. What is the yield of the stock, rounded to the nearest tenth of a percent?

Solution:

Write the yield as a fraction. Yield = $\frac{\text{Annual dividend per share}}{\text{Current price of one share}}$ or $k = \frac{D}{V}$

$$=\frac{3.40}{67.40}\approx 0.050445103 \text{ or } 5.04\%$$

The yield is about 5.04%

Formulas Related to Dividends on Preferred and Common Stock Dividend per share (preferred) = Par value × Dividend rate Total preferred dividend = Number of shares × Dividend per share Total common dividend = Total dividend – Total preferred dividend Dividend per share (common) = $\frac{\text{Total common dividend}}{\text{Number of shares (common)}}$

Example:

The board of directors of Discovery Developers, Inc. has declared a dividend of Php 53,000,000. The company has 60,000 shares of preferred stocks that pay Php 50 per share and Php 100,000 shares of common stock. Calculate the amount of dividends due the preferred shareholders and dividend per share of common stock.

Solution:

Step 1: Because the preferred dividend is stated in pesos (Php 50 per share), we skip to step 2.

Step 2:

Total preferred dividend = Number of shares \times Dividend per share

Total preferred dividend = $60,000 \times 50 = Php 3,000,000$

Step 3:

Total common dividend = Total dividend – Total preferred dividend

Total common dividend = 53,000,000 - 3,000,000 = 50,000,000

Step 4:

Dividend per share (common) = $\frac{\text{Total common dividend}}{\text{Number of shares (common)}}$

Dividend per share (common) = $\frac{50,000,000}{100,000}$

Dividend per share (common) = Php 500 per share



For more knowledge about Stocks and Basic Bonds, please check the link provided; <u>https://www.investopedia.com/articles/fundamental-analysis/11/valuation-prefered-stock.asp</u> <u>https://www.wallstreetmojo.com/common-stock-formula/</u> https://www.diffen.com/difference/Bond vs Stock

REMEMBER



APPLICATION

ACTIVITY:

Read the question and solve the problem below. Write

your complete solution.

One share of Mang Tubig preferred stocks pays an annual dividend of Php 1.60. Today Mang Tubig closed at Php 25.40 with a net change of -Php 0.50. What was the stock's yield at yesterday's closing price?



https://www.scholastic.com/teachers/articles/teaching-content/stocks-and-bonds/ https://www.thebalance.com/how-to-calculate-your-own-finance-charge-960659 https://www.input1.com/About-Input-1/News/Articles-White-Papers/Rule-of-78s