# CHAPTER 6: Logic

**Objectives:**
**a.** Determine whether or not a sentence is a statement
**b.** Identify whether a compound proposition is tatulogy or not
**c.** Construct and us truth tables to show that statements are equivalent

## Lesson 1: Logic Statement and Quantifiers

English sentences may be classified as *declarative, interrogative, exclamatory* or *imperative.* In the study of logic or mathematics, we are concerned with only one type of sentence, that is, we assume that we are able to recognize a declarative sentence (or statement) and to form an opinion as to whether it is true or false.

**Statement**

- Is a declarative sentence that can be meaningfully classified as either true or false. A statement cannot be a question, an instruction, or an opinion.

**Example:** Which of the following are statements? For each statement identified, discuss whether it is true or false.

1. Every triangle has three sides.
2. The price of a Samsung Galaxy tablet was Php 11, 000 on December 24, 2014.
3. Write a letter.
4. $y = 6$
5. What is the exchange rate from United States dollars to pesos?

**Solution:**

1. This is a statement. It happens to be true.
2. This is a statement, although few of us can say whether it is true or false.
3. This is not a statement. An instruction is never a statement.
4. This is not a statement, we are not told what y is.
5. This is not a statement. This is a question.

To avoid writing the statements in words all the time, we usually label statements with lowercase letters $p, q, r, \ldots$ If $p$ denotes the statement "Zamboanga City is located at the tip of Zamboanga Peninsula", then instead of saying that the above statement is true or false, we can simply represent value of $p$ as F.

In mathematical treatment of logic, we avoid an unclear meaning, ambiguous situations, differences of opinions by assuming a statement as either true or false.

**Compound Statements**

Is a statement formed by connecting two or more statements or by negating a single statement. It is formed by joining two propositions. The words or phrases (or symbols) used to form compound statements are called *connectives.*

**Example:**

"Karel is beautiful and Loren is selfish."

"Karel is beautiful", "Loren is selfish"; joined by the connectives *and*.

If $p$ and $q$ are statements, then the compound statements

$$\sim p,\ p \vee q,\ p \wedge q,\ \text{and}\ p \Rightarrow q$$

can be formed using the negation symbol $\sim$ and connectives $\vee$, $\wedge$ and $\Rightarrow$. These statements are read as $\text{not}\ p,\ p\ \text{or}\ q, p\ \text{and}\ q,\ \text{if}\ p\ \text{then}\ q$, respectively. We can use the truth table to specify each of these compound statements. A truth table gives the statement's truth value, *T* (true) or *F* (false), for all possible values of its variable.

The **truth value** of a statement is either true (denoted by *T)* or false (denoted by *F).*

A **truth table** is a table that shows the truth value of a compound statement for all possible cases.

**Negation**

- The negation of any simple statement can be formed by putting "not" into the statement.

**Example:**

$p$ : India is in South Asia.

$\sim p$: India is not in South Asia.

If $p$ is true, then $\sim p$ cannot also be true.

If $p$ is a statement, then the statement $\sim p$ (read as not p or the negation of p) is false if $p$ is true, and true if $p$ is false.

| $p$ | $\sim p$ |
|---|---|
| T | F |
| F | T |

The negation of $p$ is sometimes called the *denial* of p. The symbol $\sim$ is called the *negation operator.*

### Disjunction

- For two statements $p$ and $q$, $p \vee q$ means either $p$ or $q$ is true or both are true.

**Example:**

$p$ : Every square is a rhombus.

$q$ : Every square is a parallelogram.

The compound statement $p \vee q$ is

$p \vee q$ : Every square is rhombus or every square is a parallelogram.

If $p$ and $q$ are statement, then the compound statement $p \vee q$ (read as p or q or the disjunction of $p$ and $q$) it is true if $p$ is true, or if $q$ is true, or if both are true, and is false, otherwise .

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

- When we use the word "or" to mean the one and only one of the simple statements is true, we call this the *exclusive or.* The correct meaning is usually inferred from the context in which the word is used
- However, when it is important to be precise (as it often is in mathematics, business, science, etc), we must carefully distinguish between the two meaning of "or". The truth table for the exclusive "or" will be different from the table shown for disjunction.

> Let $p$ and $q$ be any statement. The exclusive disjunction of $p$ and $q$, read as "either $p$ or $q$, but not both" is denoted symbolically $p \underline{\vee} q$.
>
> We define $p \underline{\vee} q$ to be true if exactly one of the statements $p$ or $q$ is true. That is, $p \underline{\vee} q$ is true if $p$ is true and $q$ is false, or, if $p$ is false and $q$ is true. It is false, if both $p$ and $q$ are false or if both $p$ and $q$ true.

**Conjunction**

- Two simple statements are combined with the word "and".

**Example:**

$p$ : Indonesia is in Asia.

$q$ : The capital of Indonesia is Jakarta.

These can be combined to form:
Indonesia is in Asia *and* the capital of Indonesia is Jakarta.
This is written $p \wedge q$, where $\wedge$ represents the word "and".

> If $p$ and $q$ are statement, then the compound statement $p \wedge q$ (read as p and q or the conjunction of $p$ and $q$), is true if both $p$ and $q$ are true, and is false, otherwise.

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

## Quantifier

- A quantifier is a word or phrase telling "how many". It comes from the Latin word *quantos*. English quantifiers include "all", "none", "some", and "not all". The quantifiers "all", "every", and "each" are interchangeable. The quantifiers "some", "there exist(s)", and "at least one" are interchangeable.

**Example:**

All of the following statements have the same meaning:

$p$ : All students are intelligent.

$q$ : Every student is intelligent.

$r$ : Each student is intelligent.

$s$ : Any student is intelligent.

Quantifiers are words that denote the number of objects or cases referred to in a given statement.

The word *all, any,* and *every* are quantifiers which illustrate that each and every object or case satisfies the given condition.

The word *some, several, one of,* and *part of* are quantifiers which illustrate that not all but at least one object or case satisfies the given condition.

For more knowledge about Logic Statement and Quantifier, please check the link provided;
https://www.math.fsu.edu/~wooland/hm2ed/Part2Module1/Part2Module1.pdf
http://socrates.bmcc.cuny.edu/jsamuels/text/mhh-discrete-03.1.pdf

## REMEMBER

- Compound statement is a statement formed by connecting two or more statements or by negating a single statement.
- The negation of any simple statement can be formed by putting "not" into the statement.
- Disjunction For two statements $p$ and $q$ , $p \vee q$ means either $p$ or $q$ is true or both are true.
- Conjunction is two simple statements are combined with the


**ACTIVITY:** Which of the following sentences are statements? Give a short reason for your answer.

1. The sum of 15 and 17 is greater than 30.
2. The number 6 has two prime factors.
3. How are you?
4. Mathematics is difficult.
5. Listen to me, Robert.
6. The sum of all interior angles of a triangle is 180 degree.
7. Heptagon have 8 sides.
8. There is no rain without clouds.

## Lesson 2: Truth table and Tautologies

In previous lesson, truth values for negation of a statement, the conjunction of two statements and the disjunction of two statement. Each of truth tables are shown below:

| $p$ | $\sim p$ |
|---|---|
| T | F |
| F | T |

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

As another example, the relationships among a statement $p$, its negation $\sim p$, and the negation of that $\sim(\sim p)$, are shown in the table below:

Double Negation

| $p$ | $\sim p$ | $\sim(\sim p)$ |
|---|---|---|
| T | F | T |
| F | T | F |

Note that the first and the third columns have identical truth value entries. This means that $p$ and $\sim(\sim p)$ are the same logically, even though they might be grammatically different.

**Example:** If $p$ is “Robert owns a car”, then $\sim(\sim p)$ can be phrased “It is not the case that Robert doesn’t have a car.”

Besides using the connectives $\vee$, $\wedge$ and $\sim$ once to form compound statements, we can use them together to form more complex statements. The parentheses are used as grouping symbols to indicate $\vee$ and $\sim$ are applied before $\wedge$.

| $p$ | $q$ | $(p \vee \sim q) \wedge p$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | F |
| F | F | F |

To obtain the table above, list each component of the statement on the top row, to the right of any intermediate steps: there is a column underneath each component or connective. Truth values are then entered in the truth table, one step at a time. See the steps shown in below:

**Step 1:**

| $p$ | $q$ | $\sim q$ | $p \vee \sim q$ | $(p \vee \sim q) \wedge p$ |
|---|---|---|---|---|
| T | T | F | | |
| T | F | T | | |
| F | T | F | | |
| F | F | T | | |

**Step 2:**

| $p$ | $q$ | $\sim q$ | $p \vee \sim q$ | $(p \vee \sim q) \wedge p$ |
|---|---|---|---|---|
| T | T | F | T | |
| T | F | T | T | |
| F | T | F | F | |
| F | F | T | T | |

**Step 3:**

| $p$ | $q$ | $\sim q$ | $p \vee \sim q$ | $(p \vee \sim q) \wedge p$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | T | T | T |
| F | T | F | F | F |
| F | F | T | T | F |

To use truth tables to determine the truth value of a compound statement, we can work from left to right in a table. We then use the columns we have figured out to deduct the truth values of the further columns.

**Example 1:**

Construct the truth table for $\sim(p \vee q) \vee (\sim p \wedge \sim q)$

**Solution:**

We need column for
$p, q, \sim p, \sim q, p \vee q, \sim(p \vee q), \sim p \wedge \sim q$ and $\sim(p \vee q) \vee (\sim p \wedge \sim q)$

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \vee q$ | $\sim(p \vee q)$ | $\sim p \wedge \sim q$ | $\sim(p \vee q) \vee (\sim p \wedge \sim q)$ |
|---|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F | F |
| T | F | F | T | T | F | F | F |
| F | T | T | F | T | F | F | F |
| F | F | T | T | F | T | T | F |

Notice that the compound statements $\sim(p \vee q)$, $\sim p \wedge \sim q$ and $\sim(p \vee q) \vee (\sim p \wedge \sim q)$ all have the same truth table.

**Example 2:** Construct the truth table for $p \wedge (q \vee r)$.

**Solution:**

We can use the truth table when we have three propositions, $p, q$ and $r$. We can construct the truth table within 9 rows: 1 for the row of statements and 8 for the different possible combinations of the truth values of $p, q$ and $r$. After that, we use the same procedure as we would with two statements.

| $p$ | $q$ | $r$ | $q \vee r$ | $p \wedge (q \vee r)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | T | T |
| T | F | T | T | T |
| T | F | F | F | F |
| F | T | T | T | F |
| F | T | F | T | F |
| F | F | T | T | F |
| F | F | F | F | F |

**Equivalent Statement**

- If two propositions $p$ and $q$ have the same truth values in every case, the compound statements are called *logically equivalent* and we write $p \equiv q$.

- Very often two statements stated in different ways have the same meaning. For example, in law, “Mang Lito agreed and obligated to paint Joseph’s house.” has the same meaning as “It was agreed and contracted by Mang Lito that he would paint the house belonging to Joseph and that Mang Lito is therefore required to paint the aforesaid house.”

**Example 1:**

Show that $\sim(p \wedge q)$ is logically equivalent to $\sim p \vee \sim q$

**Solution:**

We will prove by comparing the truth values for both statements.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \wedge q$ | $\sim(p \wedge q)$ | $\sim p \vee \sim q$ |
| T | T | F | F | T | F | F |
| T | F | F | T | F | T | T |
| F | T | T | F | F | T | T |
| F | F | T | T | F | T | T |

First, we find the columns 3 and 4 to be easy for they have the opposite truth values of columns 1 and 2. The truth values of column 5 can be determined by comparing the columns 1 and 2 to be both true. Column 6 simply has the opposite truth value of column 5. To find the truth value of column 7, we remember that it is true when either column 3 or column 4 is true and when both column 3 and column 4 are true. Finally, we notice that the entries in column 6 and 7 of the truth table are the same; hence, the two compound statements are logically equivalent.

**Example 2:** Show that $\sim p \wedge \sim q$ is logically equivalent to $\sim(p \vee q)$.

**Solution:**

Construct the truth table.

| $p$ | $q$ | $p \vee q$ | $\sim p$ | $\sim q$ | $\sim p \wedge \sim q$ | $\sim(p \vee q)$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F |
| T | F | T | F | T | F | F |
| F | T | T | T | F | F | F |
| F | F | F | T | T | T | T |

The entries under $\sim p \wedge \sim q$ and $\sim(p \vee q)$ are the same, so the two statements are logically equivalent.

**De Morgan's Laws of Logic**

For any two statements $p$ and $q$, we have:

1. $\sim(p \vee q) \equiv \sim p \wedge \sim q$
2. $\sim(p \wedge q) \equiv \sim p \vee \sim q$

Two statements are logically equivalent (or simply equivalent) if they have identical truth values under all possible situations. The notation $p \equiv q$ is used to denote the fact that $p$ and $q$ are logically equivalent.

## Tautologies

- In logic, a statement represented by an entire column of T values is called a *tautology.* Such a proposition is *always true*, no matter what.

Consider the statement: All students study Mathematics or all students do not study Mathematics. This is a tautology, as can be shown in a truth table by considering the result of $p \vee \sim p$.

| $p$ | $\sim p$ | $p \vee \sim p$ |
|---|---|---|
| T | F | T |
| F | T | T |

Since the entries in the final column $p \vee \sim p$ are all true, this is a tautology.

In the simplest tautology, we may replace the symbol $p$ with any simple statement, whether its truth values are known or not.

For Example**:**

$p \vee \sim p$ : A number is even or a number is not even.

$p \vee \sim p$ : A square is a triangle or a square is not a triangle.

$p \vee \sim p$ : It will rain or it will not rain.

$p \vee \sim p$ : $x+3=5$ or $x+3 \neq 5$

A tautology is a compound statement $p(p,q,...)$ that is true in every possible case.

**Example:**

Show that $(p \wedge q) \vee (\sim p \wedge \sim q)$ is a tautology.

**Solution:**

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \vee q$ | $\sim p \wedge \sim q$ | $(p \wedge q) \vee (\sim p \wedge \sim q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | T |
| T | F | F | T | T | F | T |
| F | T | T | F | T | F | T |
| F | F | T | T | F | T | T |

All entries in the final column $(p \wedge q) \vee (\sim p \wedge \sim q)$ are true; hence, the statement is a tautology.

For more knowledge about Truth tables and tautologies, please check the link provided;
https://study.com/academy/lesson/tautology-in-math-definition-examples.html
https://www.whiteplainspublicschools.org/cms/lib/NY01000029/Centricity/Domain

## REMEMBER

- **Equivalent Statement-** If two propositions $p$ and $q$ have the same truth values in every case, the compound statements are called *logically equivalent* word "and".
- In logic, a statement represented by an entire column of T values is called a *tautology.* Such a proposition is *always true*, no matter what.
- **De Morgan's Laws of Logic**

For any two statements $p$ and $q$, we have:

$$\sim (p \vee q) \equiv \sim p \wedge \sim q$$

$s$


**ACTIVITY:** Write each statement in symbolic form, using the given symbols.

Let p: I will study.
Let q: I will pass the test.
Let r: I am foolish.

1. I will study or I am foolish.
2. I will study or I will not pass the test.
3. I will study and I will pass the test.
4. I will past the test or I am foolish.
5. I am not foolish and I will pass the test.
6. I will not study or I am foolish.
7. I will study or I will not study.
8. I will study and I will pass the test, or I am foolish.

# Lesson 3: The Conditional and Biconditional

## Conditional Statement

One of the most important ways to combine two statements is by the condition-consequence linkage, something called the *"if-then"* form. The parts of the conditional *if p, then q* can be identified by name:

- ***p*** is called the *premise, hypothesis,* or the *antecedent*. It is an assertion or a statement that begins our argument. The antecedent is usually preceded by the word *if.*
- ***q*** is called the *conclusion* or the *consequent*. It is an ending or a statement that closes our argument. The consequent is usually preceded by the word *then.*

Let us consider the following conditional statement.
"If Helen finishes her homework, then she will clean her room."

There are different ways to write the conditional *if p, then q*. notice that the hypothesis *p* is connected to the word *if* in the samples known:

$p \rightarrow q$ : If Helen finishes her homework, then she will clean her room.
(Antecedent or hypothesis: Helen finishes her homework; Consequent or conclusion: she will clean her room.)

$p \rightarrow q$ : Helen will clean her room if she finishes her homework.
(conclusion: Helen will clean her room; hypothesis: she finishes her homework.)

The hypothesis and the conclusion of a conditional statement can have a truth value of true or false, as can the conditional statement itself. First of all, observe that the given conditional statement is made up of two simple propositions:

$p$ : Helen finishes her homework.

$q$ : Helen will clean her room.

**Four cases to consider in conditional statement:**

1. $p$ and $q$ might both be true.
2. $p$ might be true and $q$ false.
3. $p$ might be false and $q$ true.
4. $p$ and $q$ might both be false.

Truth table for Conditional Statement.

| $p$ | $q$ | $p \Rightarrow q$ |
|---|---|---|
| T | T | T |

| | | |
|---|---|---|
| T | F | F |
| F | T | T |
| F | F | T |

Take note:

1. A conditional statement is false only when the hypothesis is true and its conclusion is false.
2. When a hypothesis is false, the conditional statement will always be considered true, whether the conclusion is true or false.

We denote the conditional connective symbolically by $\Rightarrow$ and the implication *If p, then q* by $p \Rightarrow q$. In the implication $p \Rightarrow q$, *p* is called the *hypothesis* or *premise* and *q* is called the *conclusion.* The implication $p \Rightarrow q$ can also be read as follows:

1. $p$ implies $q$
2. $p$ only if $q$
3. $q$ if $p$
4. $p$ is sufficient for $q$
5. $q$ is necessary for $p$

| $p$ | $q$ | $\sim p$ | $\sim p \vee q$ | $p \Rightarrow q$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

**Biconditional Statement**

When a conditional statement and its converse are both true, we can write them as a single biconditional statement.

A *biconditional statement,* or simply a *biconditional*, is a statement that contains the phrase "*if and only if". * Any valid definition can be written as a biconditional statement.

For example:

$p$ : Jill is happy. $q$ : Jack is attentive.

When we say "Jill is happy if and only if Jack is attentive.", we mean that if Jill is happy, then Jack is attentive, and if Jack is attentive then Jill is happy.

In symbols, we would write this as:

$$(p \Rightarrow q) \wedge (q \Rightarrow p)$$

Biconditional statement is denoted by the symbol $\Leftrightarrow$. The compound proposition

$p \Leftrightarrow q$ ( $p$ if and only if $q$ ) is equivalent to $(p \Rightarrow q) \wedge (q \Rightarrow p)$

$p \Leftrightarrow q$ can be read as:

1. $p$ is necessary and sufficient for $q$
2. $p$ implies $q$ and $q$ implies $p$

Truth table for Conditional Statement.

| $p$ | $q$ | $p \Rightarrow q$ | $q \Rightarrow p$ | $(p \Rightarrow q) \wedge (q \Rightarrow p)$ | $p \Leftrightarrow q$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | F | T | F | F |
| F | T | T | F | F | F |
| F | F | T | T | T | T |

**Example:**

Let $p$ and $q$ denote the following propositions:

$p$ : Joseph is happy. $q$ : Joseph is not studying.

Then, “A *necessary condition* for Joseph to be happy is that Joseph is not studying” means “If Joseph is happy, he is not studying.” Moreover, “A *sufficient condition* for Joseph to be happy is that Joseph is not studying.” means “If Joseph is not studying, he is happy.”

If both implications $(p \Rightarrow q, q \Rightarrow p)$ are true statements, then the biconditional is true.

In other words, Joseph is happy if and only if he is not studying.

For more knowledge about Conditional and Biconditional Statement, please check the link provided;
https://www.zweigmedia.com/RealWorld/logic/logic3.html
https://www.slideshare.net/dannahpaqz/conditional-and-biconditional-statements
https://www.varsitytutors.com/hotmath/hotmath_help/topics/conditional-statements

## REMEMBER

- One of the most important ways to combine two statements is by the condition-consequence linkage, something called the *"if-then"* form.
- A *biconditional statement,* or simply a *biconditional*, is a statement that contains the phrase "*if and only if". Any valid* definition can be written as a biconditional statement.


**ACTIVITY:**
For each given sentence: (a) Identify the hypothesis *p*. (b) identify the conclusion *q*.

1. If it rains, then the game is cancelled.
2. If it is 9:05 am, then I'm late to class.
3. The perimeter of a square is 4x+8 if one side the square is x+2.
4. If a polygon has exactly three sides, it is a triangle.
5. If a polygon is regular, then all of its sides are congruent.

# Lesson 4: Related Conditionals

Unlike conjunction and disjunction, the order of the two simple statements in a conditional statement makes a difference.

"If a figure is a square, then it is a rectangle." says something quite different from "If a figure is a rectangle, then it is a square."

Conditionals involving the negations of the hypothesis and the conclusion can complicate matters further. Since these forms and the relationships among them are often useful, it is helpful to have special terms to distinguish one from another.

As we read through the following definitions, it might help us think of a simple example of a conditional. For instance,

$p$ : My shoes are too small. $q$ : My feet hurt.

Then, the original conditional $p \Rightarrow q$ is:

"If my shoes are too small, then my feet hurt."

## Converse

- The converse is formed by exchanging the hypothesis and conclusion of the conditional.

  (If my feet hurt, then my shoes are too small.)

The implication *if q, then p* is called the *converse* of the implication *if p, then q.* that is $q \Rightarrow p$ is the converse of $p \Rightarrow q$.

| **Truth table for implication $p \Rightarrow q$ and its converse $q \Rightarrow p$** | | | |
|---|---|---|---|
| $p$ | $q$ | $p \Rightarrow q$ | $q \Rightarrow p$ |
| T | T | T | T |
| T | F | F | T |
| F | T | T | F |
| F | F | T | T |

Notice that $p \Rightarrow q$ and $q \Rightarrow p$ are not equivalent.

**Inverse**

- The inverse is formed by negating both the hypothesis and conclusion of the conditional.
  (If my shoes are not too small, then my feet do not hurt.)

The implication *If not p, then not q,* written as $\sim p \Rightarrow \sim q$, is called the *inverse* of the implication *If p, then q.*

**Contrapositive**

- The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.
  (If my feet do not hurt, then my shoes are not too small.)

The implication *If not q, then not p,* written as $\sim q \Rightarrow \sim p$, is called the *contrapositive* of the implication $p \Rightarrow q$.

| **Truth tables for** $p \Rightarrow q, q \Rightarrow p$, $\sim p \Rightarrow \sim q$ **and** $\sim q \Rightarrow \sim p$ | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Statements** | | **Implication** | **Converse** | | | **Inverse** | **Contrapositive** |
| $p$ | $q$ | $p \Rightarrow q$ | $q \Rightarrow p$ | $\sim p$ | $\sim q$ | $\sim p \Rightarrow \sim q$ | $\sim q \Rightarrow \sim p$ |
| T | T | T | T | F | F | T | T |
| T | F | F | T | F | T | T | F |
| F | T | T | F | T | F | F | T |
| F | F | T | T | T | T | T | T |

**Example:**

Universal statement: “All dogs are animals.”

*Conditional form:* “If something is a dog, then it is an animal.”

*Converse:* “If something is an animal, then it is a dog.”

*Inverse:* “If something is not a dog, then it is not an animal.”

"Anything that is not a dog is not an animal."

*Contrapositive:* "If something is not an animal, then it is not a dog."

"Anything that is not an animal is not a dog."

For more knowledge about Converse, Inverse and Contrapositive Statement, please check the link provided;

https://www.varsitytutors.com/hotmath/hotmath_help/topics/converse-inverse-contrapositive

https://tutors.com/math-tutors/geometry-help/converse-inverse-contrapositive

## REMEMBER

- The converse is formed by exchanging the hypothesis and conclusion of the conditional.
- The inverse is formed by negating both the hypothesis and conclusion of the conditional.
- The contrapositive is formed by negating both the hypothesis and the conclusion of the converse of the conditional.


**ACTIVITY:**
Write each sentence in symbolic form, using the given symbols.

p: The test is easy.
q: Joseph studies.
r: Joseph passes the test.

1. If the test is easy, then Joseph will pass the test.
2. If Joseph studies, then Joseph will pass the test.
3. If the test is not easy, then Joseph will not pass the test.
4. The test is easy if Joseph studies.
5. Joseph will not pass the test If Joseph doesn't study.
6. Joseph passes the test if the test is easy.

## Lesson 5: Valid Reasoning

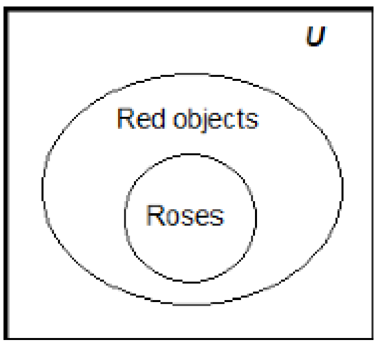
In problem solving, the reasoning used is said to be *valid* if the conclusion follows unavoidably from the hypotheses. Consider the following example:

Hypotheses: All roses are red.

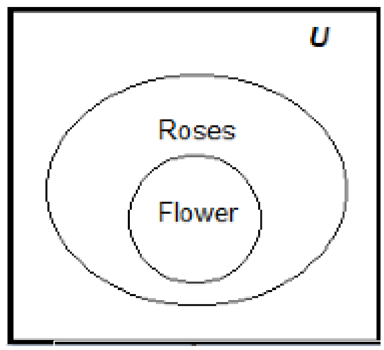
This flower is a rose.

Conclusion: therefore, this flower is red.

The statement "All roses are red" can be written as implication, "If a flower is a rose, then it is red" and pictured with the Venn diagram below:




a. All roses are red.




b. The flower is a rose.

Thus, the reasoning is valid because it is impossible to draw a picture satisfying the hypotheses and contradicting the conclusion.

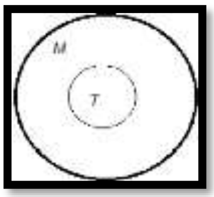
Consider the following argument.

Hypotheses: All mathematics teachers are mathematicians.

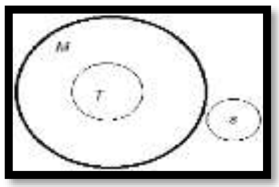
Some mathematicians are not student.

Conclusion: Therefore, no mathematics teacher is a student.

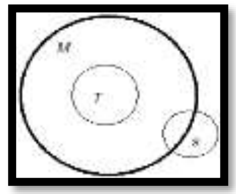
Let $T$ be the set of mathematics teachers, $M$ be the set of mathematicians, and $S$ be the set of students. Then the statement, "All mathematics teachers are mathematicians" can be pictured in the figure a. The statement "some mathematicians are not students" can be pictured in several ways but three of these are illustrated in figure b, figure c and figure d.



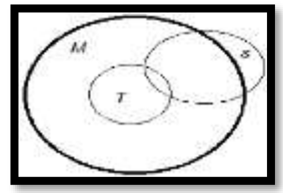

a.




b.




c.




d.

(a). All mathematics teachers are mathematicians.

(b)-(d). Some mathematicians are not students.

According to figure d, it is possible that some mathematics teachers are students, and yet the given statements are satisfied. Therefore, the conclusion that "No mathematics teacher is a student" does not follow from the given hypotheses. Hence, the reasoning is not valid.

If a single picture can be drawn to satisfy the hypotheses of an argument and contradict the conclusion, the argument is not valid. However, to show that an argument is valid, all possible pictures must be considered to show that there are no contradictions. There must be no way to satisfy the hypotheses and contradict the conclusion if the argument is valid.

**Example:** As stated in lesson 2, tautologies are used to develop strong arguments. For example, two statements are made:

$r \Rightarrow m$ : If it rains, then, we will watch movies.

$r$. It rains.

What can we conclude?

**Solution:**

Combine the two statements as a conjunction. This compound statement will be used as a hypothesis of a conditional.

The reasoning becomes: If $r \Rightarrow m$ and $r$, then $m$.

Written symbolically: $\left[(r \Rightarrow m) \wedge r\right] \Rightarrow m$

Therefore, we conclude that $m$: We will watch the movies.

Let us test the reasoning in the truth table, working from the innermost parentheses first. Since as shown in the last column, the statement is always true, this is tautology.

| $r$ | $m$ | $r \Rightarrow m$ | $(r \Rightarrow m) \wedge r$ | $\left[(r \Rightarrow m) \wedge r\right] \Rightarrow m$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | F | T | F | T |

From example, we know that to show a statement is true, we must show it is true for all cases. We can show that a proposition is false, by simply finding a counterexample to show that the proposition is false.

**Law of Detachment**

If the implication $p \Rightarrow q$ is true and *p* is true, then *q* must be true.

Note: $\left[(r \Rightarrow m) \wedge r\right] \Rightarrow m$ is a form of argument called the Law of Detachment.

**Example:**

Make a conclusion based on the two given hypotheses.

Hypothesis 1:

$p$ : All multiples of 20 are multiples of 5.

Hypothesis 2:

$q$ : 60 is a multiple of 20.

**Solution:** Conclusion: 60 is a multiple of 5.

For more knowledge about Valid reasoning and Law of Detachment, please check the link provided;
https://study.com/academy/lesson/law-of-detachment-in-geometry-definition-examples.html
https://courses.lumenlearning.com/atd-pima-philosophy/chapter/1-2-arguments-types-of-reasoning/

## REMEMBER

- In problem solving, the reasoning used is said to be *valid* if the conclusion follows unavoidably from the hypotheses.
- **Law of Detachment**
  - If the implication $p \Rightarrow q$ is true and *p* is true, then *q* must be true.


**ACTIVITY:**
Determine if the following argument is valid:

a. Hypotheses: If you eat spinach, then you will be strong.
You eat spinach.
Conclusion: Therefore, you will be strong.

b. Hypotheses: If Carlos goes skating, he will break his legs.
If Carlos break his leg, he cannot enter the contest.
Carlos gets skating.
Conclusion: Therefore, Carlos cannot enter the dance contest.


http://web.mnstate.edu/peil/geometry/Logic/4logic.htm
https://www.ck12.org/geometry/converse-inverse-and-contrapositive-statements/lesson/Converse-Inverse-and-Contrapositive-GEOM/