CHAPTER 1: Nature of Mathematics

Objectives:
a. Identify and differentiate Patterns in Nature.
b. Understand the Fibonacci Sequence.
c. Appreciate the beauty of Mathematics in terms of Patterns and Number in Nature and in the World.

Lesson 1: Patterns and Numbers in Nature and in the World

Patterns in Nature

Patterns in nature are visible regular forms found in the natural world. The patterns can sometimes be modeled mathematically and they include symmetries, trees, spirals, meanders, waves, foams, tessellations, cracks and stripes.

Mathematics, physics and chemistry can explain patterns in nature at different levels. Patterns in living things express the underlying biological processes. Studies of pattern formation make use of computer models to simulate a wide range of patterns.

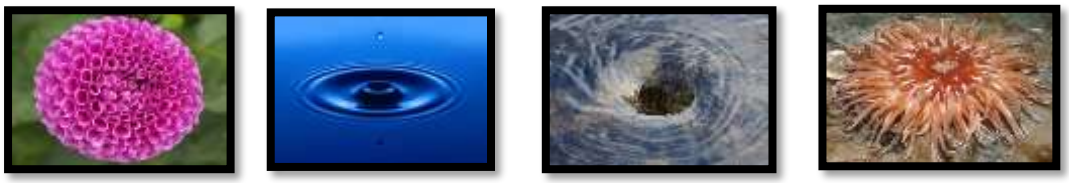
Types of Pattern in Nature

1. **Symmetry**
   - is when different sides of something are alike. These reflections may be mirror images with only two sides, like the two sides of our bodies; they may be symmetrical on several sides, like the inside of an apple sliced in half; or they might be symmetrical on all sides, like the different faces of a cube.

Types of Symmetry

- **Bilateral or Mirror Symmetry** is symmetry with respect to reflection. That is, a figure which does not change upon undergoing a reflection has reflectional symmetry.

- **Rotational symmetry or radial symmetry** is also found at different scales among non-living things including the crown-shaped splash pattern formed when a drop falls into a pond, and both the spheroidal shape and rings of a planet like Saturn.



- **Fivefold symmetry** is found in the echinoderms, the group that includes starfish, sea urchins, and sea lilies. The reason for the fivefold (penta-radiate) symmetry of the echinoderms is puzzling.

- **Six-fold symmetry**: each flake's structure forming a record of the varying conditions during its crystallization, with nearly the same pattern of growth on each of its six arms.

Note: Crystals in general have a variety of symmetries and crystal habits; they can be cubic or octahedral, but true crystals cannot have fivefold symmetry (unlike quasicrystals).

2. Fractals

- are infinitely self-similar, iterated mathematical constructs having fractal dimension. Infinite iteration is not possible in nature so all 'fractal' patterns are only approximate.

3. Spirals

- is a curve which emanates from a point, moving farther away as it revolves around the point.




4. Tessellations

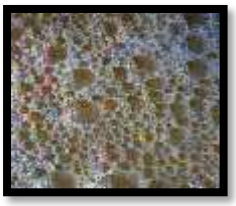
- are patterns formed by repeating tiles all over a flat surface. There are 17 wallpaper groups of tilings. While common in art and design, exactly repeating tilings are less easy to find in living things.




5. Bubbles/ Foams

- A soap bubble forms a sphere, a surface with minimal area — the smallest possible surface area for the volume enclosed.






6. Stripes

- These patterns have an evolutionary explanation: they have functions which increase the chances that the offspring of the patterned animal will survive to reproduce.




7. **Cracks**
   - are linear openings that form in materials to relieve stress. Thus, the pattern of cracks indicates whether the material is elastic or not.




For more knowledge about Patterns in Nature, please check the link provided;
https://study.com/academy/lesson/patterns-in-nature-definition-examples.html

ACTIVITY: Capture the Beauty of Mathematics
For this exercise, go around your environment and capture at least 10 different things that has a pattern. Compile it and paste in a bond paper, compare your answer with your seatmate. 10 students will present the captured patterns in front of the class.

Lesson 2: Fibonacci Sequence

Sequence

- Is an ordered list of numbers, called ***terms*** that may have repeated values. The arrangement of these terms is set by a definite rule.

Example:

Analyze the given sequence for its rule and identify the next three terms.

a. $1,10,100,1000,...$
b. $2,5,9,14,20,...$

Solution:

a. By looking at the set of number, you will observe that the sequence is power by 10. Following the rule, the 5^{th} number= 10000, 6^{th} number= 100000 and the 7^{th} number= 1000000.

b. Looking at the set of number, you will observe that the sequence get by adding the difference of two previous number to the last listed next number. Following the rule, (2 and 5) 3, (5 and 9) 4, (9 and 14) 5, and (14 and 20) 6. Now to get

the next three terms we should use 7, 8, and 9, respectively. 6th number= 27, 7th number= 35, and 8th number=44.

Fibonacci Sequence

The Fibonacci Sequence is the series of numbers:

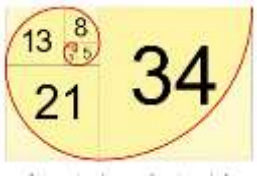
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

The next number is found by adding up the two numbers before it.

- The 2 is found by adding the two numbers before it (1+1)
- The 3 is found by adding the two numbers before it (1+2),
- And the 5 is (2+3),
- and so on!

Fibonacci is found in a spiral, look the image below:

When we make squares with those widths, we get a nice spiral:



Do you see how the squares fit neatly together?
For example 5 and 8 make 13, 8 and 13 make 21, and so on.

This spiral is found in nature!

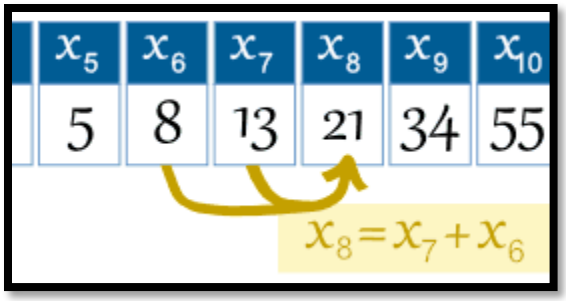
Rule in Fibonacci Sequence

First, the terms are numbered from 0 onwards like this:

n =	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	...
x_n =	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	...

So term number 6 is called x_6 (which equals 8).

Example: the **8th** term is the **7th** term plus the **6th** term: $x_8 = x_7 + x_6$



So we can write the rule:

The Rule is $X_n = X_{n-1} + X_{n-2}$

where:

- X_n is term number "n"
- X_{n-1} is the previous term $(n-1)$
- X_{n-2} is the term before that $(n-2)$

Example: term 9 is calculated like this:

$$X_9 = X_{9-1} + X_{9-2}$$
$$= X_8 + X_7$$
$$= 21 + 13$$
$$= 34$$

REMEMBER

- Patterns all over the environment is related to mathematics. We have different patterns that can be seen everywhere. We must appreciate the beauty of Mathematics.
- Here is the Fibonacci sequence again:

$n =$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	...
$X_n =$	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	...

There is an interesting pattern:

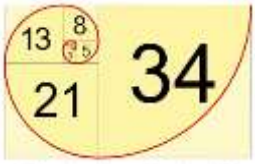
- Look at the number $X_3 = 2$. Every **3**rd number is a multiple of **2** $(2,\ 8,\ 34,\ 144,\ 610,\ ...)$
- Look at the number $X_4 = 3$. Every **4**th number is a multiple of **3** ($(3,\ 21,\ 144,\ ...)$
- Look at the number $X_5 = 5$. Every **5**th number is a multiple of **5** ($(5,\ 55,\ 610,\ ...)$

And so on (every **n**th number is a multiple of **(** X_n).

For more knowledge about Fibonacci Sequence, please check the link provided;
https://study.com/academy/lesson/fibonacci-numbers-in-nature-lesson-for-kids.html

ACTIVITY: Fill the Spiral

For exercise, continue the given spiral by finding the Fibonacci sequence. Write your solution in a one sheet of paper. Take note, stop only when you get the number of n=30.



https://digitalcommons.liberty.edu/cgi/viewcontent.cgi?referer=https://www.google.com/&httpsredir=1&article=1347&context=honors
Soaring 21st Century Mathematics